12.808 Handout : Wind driven circulation

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1 Sverdrup Flow

Our goal is to understand how wind stress at surface might drive sub-surface flow on *long* time scales (much longer than a day). The strategy is to start with the simplest possible set of equations and see if we can learn something from that.

1. Start with these *inviscid* equations (subscripts represent derivatives, superscripts represent direction)

$$u_t + uu_x + vu_y + wu_z - fv = -\frac{1}{\rho_0}p_x + \tau_z^x \tag{1}$$

$$v_t + uv_x + vv_y + wv_z + fu = -\frac{1}{\rho_0}p_y + \tau_z^y \tag{2}$$

$$0 = -\frac{1}{\rho}p_z \quad - g \tag{3}$$

$$u_x + v_y \quad + w_z = 0 \tag{4}$$

2. We are interested in dynamics that happen on a time scale $\sim$ a year. What term can you drop from the equations? What scaling argument can you make for this?

3. We want to drop the non-linear terms to keep things simple. When is this valid? Compare the non-linear terms to the Coriolis terms.

4. Now, our forcing is *Ekman pumping* at the base of the Ekman layer. What can you say about stress τ felt by water parcels just below the Ekman layer? Can you simplify our set of equations even more?

5. Write down the simplified set of equations after making all these assumptions. What is this momentum balance called?

6. We would like to reduce the equations we are working with. The strategy is to combine the momentum and continuity equations. Remember that we are taking into account a spherical Earth by saying that $f = f_0 + \beta y$. This is called a β plane approximation.

7. Great! Now to understand this equation better, we will vertically integrate it from the bottom $(z = -D)$ to the base of the Ekman layer $(z = \delta)$. This will give us an expression for *depth-integrated* meridional (north-south) transport.

   Assume a flat bottomed, rectangular ocean. What is w at the bottom? This is the *bottom boundary condition*. What is w at the base of the Ekman layer?

8. Are you convinced that the wind stress, that has no *direct* impact on water below the Ekman layer, can drive a substantial meridional flow by *Ekman pumping*?

2 PV conservation

- Start with

$$q = \frac{f + \zeta}{h}$$

  This is valid if you're thinking about a constant density fluid that has an aspect ratio like the ocean on a spherical Earth.

- Imagine a fluid parcel magically constrained to move at a single latitude that has $\zeta = 0$ initially. Imagine now that this parcel is magically compressed to half it's height. Write down it's initial PV. What is the final value of PV? What changes?

  Since PV is conserved, final PV f_1/h_1 = initial PV, f_0/h_0 so,

$$f_1 = h_1 \frac{f_0}{h_0} = f_0/2$$

  Since, the parcel must halve it's f

- What if it's height was instead doubled?

- For a spherical Earth, write an appropriate approximation to f. What is this called?

- Assume instead that the constraint was that $\zeta = 0$ always and some flow moved the parcel to a latitude twice it's initial one. What happens to the parcel's height?

- Given the approximations we made in the earlier section, compare the two terms in the numerator of q to each other using $\zeta = v_x - u_y$. Scale the terms and see if one of our earlier approximations let's us simplify the equation.

- Now we can use this simplified equation for q and a nice diagram to physically explain Sverdrup balance.